Some properties of *k*-neighborhood template 𝒜 -type three-dimensional bounded cellular acceptor

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1. Introduction and Preliminaries

In the multi-dimensional pattern processing, designers often use a strategy whereby features are extracted by projecting high-dimensional space on low dimensional space. On the other hand, due to the advances in many application areas such as computer animation, dynamic image processing, and so on, the study of four-dimensional pattern processing has been of crucial importance. Thus, the study of four-dimensional automata as the computational models of four-dimensional pattern processing has been meaningful. So, from this viewpoint, we introduce a computational model, *k*-neighborhood template 𝒜-type three-dimensional bounded cellular acceptor (abbreviated as 𝒜-3*BCA*(*k*)) on four-dimensional tapes, and show some properties. An 𝒜 -3*BCA*(*k*) consists of a pair of a converter and a configuration-reader. The former converts the given four-dimensional tape to three-dimensional configuration[2]. The latter determines whether or not the derived three-dimensional configuration is accepted, and concludes the acceptance or non-acceptance of given four-dimensional tapes. When a four-dimensional input tape is presented to the 𝒜 -3*BCA*(*k*), a three-dimensional cellular automaton as the converter first reads it along the time axis. From this process, the four-dimensional tape is converted to a configuration of the converter which is a state matrix of a three-dimensional cellular automaton. Second, three-dimensional automaton as the configuration-reader reads the configuration and determines its acceptance. We say that an input four-dimensional tape is accepted by the 𝒜 -3*BCA*(*k*) if and only if the configuration is accepted by the configuration-reader. Therefore, the accepting power of the 𝒜 -3*BCA*(*k*) depends on how to combine the converter and the configuration-reader. An 𝒜 -3*DBCA*(*k*) (𝒜 -3*NBCA*(*k*)) is called a *k*-neighborhood template 𝒜-type three-dimensional deterministic bounded cellular acceptor (*k*-neighborhood template 𝒜-type three-dimensional nondeterministic bounded cellular acceptor).

This paper investigates how the difference of the neighborhood template of the converter affects the accepting powers of 𝒜 -3*BCA*(*k*)'s. In general, it is well-known that two-dimensional digital pictures have 4- and 8-connectedness, and three-dimensional digital pictures have 6- and 26-connectedness. However, we include the remarkable pixel or voxel in neighbor. In other words, we deal with 5- and 9-connectedness in the two-dimensional case, and 7- and 27-connectedness in the three-dimensional case in this paper. In Addition, we also investigate how the difference of configuration-reader affects the accepting powers of 𝒜-3*BCA*(*k*)'s. A configuration-reader *DA* (*NA*, *DB*, *NB*, *DO*, *NO*, *DOP*, *NOP*, *DP*, *NP*, *DTM*, *NTM*) is called a three-dimensional deterministic finite automaton (three-dimensional nondeterministic finite automaton, deterministic three-dimensional bounded cellular acceptor, nondeterministic three-dimensional bounded cellular acceptor, three-dimensional deterministic on-line tessellation acceptor, three-dimensional nondeterministic on-line tessellation acceptor, deterministic three-way parallel/sequential array acceptor, nondeterministic three-way parallel/sequential array acceptor, deterministic four-way parallel/sequential array acceptor, nondeterministic four-way parallel/sequential array acceptor, three-dimensional deterministic Turing machine, three-dimensional nondeterministic Turing machine)[1]. Let *T*(*M*) be the set of four-dimensional tapes accepted by a machine *M*, and let ℒ[𝒜-3*DBCA*(*k*)] = {*T*|=*T*(*M*) for some 𝒜-3*DBCA*(*k*) *M*}. ℒ[𝒜-3*NBCA*(*k*)], etc. are defined in the same way as ℒ[𝒜-3*DBCA*(*k*)].

2. Main Results

We mainly deal with only four-dimensional input tapes which each side-length is equivalent. Then, we get the following theorems.

Theorem 1. *For each* 𝒜 ∈ {*DA*, *NA*, *DB*, *NB*, *DO*, *NO*, *DOP*,*NOP*, *DP*, *NP*, *TM*} *and for each X* ∈{*D*, *N*}, ℒ[𝒜-3*XBCA*(1)] ⊊ ℒ[𝒜-3*XBCA*(7)].

Theorem 2. *For each* 𝒜 ∈ { *DA*, *NA*, *DB*, *NB*, *DO*, *NO* }, ℒ[𝒜-3*DBCA*(7)] ⊊ ℒ[𝒜-3*DBCA*(27)].

Theorem 3. *For each* 𝒜 ∈ { *DA*, *NA*, *DB*, *NB*, *DO*, *NO*, *DOP*, *NOP*, *DP*, *NP*, *TM* }, ℒ[𝒜-3*DBCA*(7)] = ℒ[𝒜-3*NBCA*(27)].

Theorem 4. *For each k* ∈ {1, 7, 27},

(1) ℒ[*DA*-3*DBCA*(*k*)] ⊊ ℒ[*NA*-3*DBCA*(*k*)] ⊊ ℒ[*NB*-3*DBCA*(*k*)] = ℒ[*NO*-3*DBCA*(*k*)] ⊊ ℒ[*NOP*-3*DBCA*(*k*)] ⊊ ℒ[*NP*-3*DBCA*(*k*)],

(2) ℒ[*DB*-3*DBCA*(*k*)] ⊊ ℒ[*NB*-3*DBCA*(*k*)],

(3) ℒ[*DO*-3*DBCA*(*k*)] ⊊ ℒ[*NO*-3*DBCA*(*k*)],

(4) ℒ[*DB*-3*DBCA*(*k*)] ⊊ ℒ[*DOP*-3*DBCA*(*k*)] ⊊ ℒ[*DP*-3*DBCA*(*k*)], *and*

(5) ℒ[*DO*-3*DBCA*(*k*)] ⊊ ℒ[*DOP*-3*DBCA*(*k*)] ⊊ ℒ[*NOP*-3*DBCA*(*k*)].

Theorem 5. *For each k* ∈ {1, 7, 27}, ℒ[*DA*-3*NBCA*(*k*)] = ℒ[*NA*-3*NBCA*(*k*)] = ℒ[*DB*-3*NBCA*(*k*)] = ℒ[*NB*-3*NBCA*(*k*)] = ℒ[*DO*-3*NBCA*(*k*)] = ℒ[*NO*-3*NBCA*(*k*)].

References

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